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AN INVESTIGATION OF STRESSES AND STRAINS IN AN INTERNALLY PRESSURIZED, COMPOSITE-JACKETED, STEEL CYLINDER

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This report presents the results of a theoretical and experimental investigation of the effects on bore strain when a portion of the wall thickness of a steel cylinder is replaced with organic composite material. The pressure vessel is pressurized internally and the bore strain is theoretically predicted and experimentally measured as a function of wall ratio. Theoretical results for various ratios of steel to composite, from all-steel to all-composite, are given. The theoretical solution was obtained by equating the
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hoop strain of the steel with the hoop strain of the composite at the material interface. Lamé's stress solution was used as input for the steel hoop strain, whereas Lekhnitskii's stress solution obtained for orthotropic cylinders was used as input to the composite hoop strain equation. The experimental results are for steel liners that have two thicknesses and are wrapped in the circumferential direction with a graphite-bismaleimide organic composite. Also presented are predicted weight savings achieved by replacing steel with the organic composite. The results show that a penalty is paid in wall thickness, but that a weight savings is achieved when a part of the steel cylinder is replaced with an organic composite.

Key words

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NOMENCLATURE

a	=	inner radius of steel liner and all-steel cylinder
A	=	material parameter from hoop strain equivalence condition
b	=	outer radius of steel liner and inner radius of composite jacket
b_o	=	outer radius of all-steel cylinder
B	=	material parameter from hoop strain equivalence condition
c	=	outer radius of composite jacket
E_r, E_θ, E_z	=	engineering moduli of composite jacket
E_s	=	engineering modulus of steel
F	=	density ratio ρ_s/ρ_j
K	=	orthotropic material parameter
p	=	pressure at $r = a$ (internal pressure of liner)
q	=	pressure at $r = b$ (external pressure of liner and internal pressure of jacket)
R	=	W_l/W_s, ratio of liner wall ratio to all-steel cylinder wall ratio
W_j	=	wall ratio of composite jacket, c/b
W_{jl}	=	wall ratio of composite jacket and steel liner, c/a
W_l	=	wall ratio of steel liner, b/a
W_s	=	wall ratio of all-steel cylinder, b_o/a
WR	=	weight reduction
$WGHT_{jl}$	=	weight of composite jacket and steel liner
$WGHT_s$	=	weight of all-steel cylinder
δ_r	=	radial displacement
ϵ_θ	=	strain in hoop direction
$\nu_{r\theta}$	=	Poisson's ratio of composite (r stress producing θ strain) (also $\nu_{rz}, \nu_{\theta z}, \nu_{z\theta}$)
ν_s	=	Poisson's ratio of steel

ρ_j = density of composite

ρ_s = density of steel

$\sigma_r, \sigma_\theta, \sigma_z$ = stress in radial, hoop, and axial directions

Subscripts:

j = jacket (composite)

l = liner (steel)

r,θ,z = radial, hoop, and axial directions

s = steel

INTRODUCTION

Organic composites have become familiar structural components in many applications that require high stiffness and low weight. This situation also exists in a current problem with Army cannon. The Army would like to design longer cannon and maintain the inertial characteristics of the shorter cannon. This accomplishment would allow current cannon mounts to be used. The longer cannon is expected to achieve higher muzzle velocity and greater accuracy than the standard cannon. The design under consideration is to replace a portion of the steel wall thickness with an organic composite (graphite fiber and a bismaleimide matrix). A previous report (ref 1) indicates that the bismaleimide composite used in conjunction with a steel liner suffers little degradation in properties when tested after experiencing temperatures of 650°F (343°C) for up to two hours. The steel liner maintains the tube projectile interface and shields the composite from the extremely hot gases. The steel also has elastic properties in the radial direction that are better than the composites for transferring loads. Indeed, O'Hara (ref 2) has shown that thick-walled composite cylinders with high ratios of hoop to radial stiffness (E_θ/E_r) are much more limited in the pressure they can contain than an isotropic cylinder of similar wall ratio. This report presents the stress-strain relationship for an organic composite-jacketed steel cylinder subjected to an internal pressurization cycle.

[1]M. A. Scavullo, M. D. Witherell, K. Miner, T. E. O'Brien, and W. Yaiser, "Experimental and Analytical Investigation of a Steel Pressure Vessel Overwrapped With Graphite Bismaleimide," ARCCB-TR-87013, Benet Weapons Laboratory, Watervliet, NY, May 1987.

[2]G. P. O'Hara, "Some Results on Orthotropic High Pressure Cylinders," ARCCB-TR-87015, Benet Weapons Laboratory, Watervliet, NY, June 1987.

In this report, the word "jacket" corresponds to the composite jacket, the term "liner" corresponds to the steel cylinder that is jacketed, and the term "compound cylinder" identifies the composite-jacketed steel cylinder.

The experimental results were obtained on cylinders having an inner diameter (ID) of 1.8 inches (4.6 cm). They are compared to the analytical results which are presented as strain as a function of overall wall ratio W_{jl} (wall ratio of composite jacket and steel liner). Results for all-steel and all-composite cylinders are presented to show the dramatic effect of using a steel liner when compared to an all-composite cylinder and to give design information for replacing the all-steel cylinder. Also discussed is the weight savings that can be achieved.

APPARATUS AND TEST SPECIMEN

The experimental investigation was conducted on internally pressurized cylinders with end closures. The end closures hold the seals and electrical feeds. Figure 1 is a schematic of the cylinder installed in the press. The figure shows a gap between the end closures and the cylinder, hence, there are no external axial (longitudinal) forces introduced into the cylinder other than those that may exist from the seal-cylinder interface. The end closures are equipped to allow the internal pressurization of the cylinder and the external monitoring of the internal strain gages. The bore strain gages were mounted to measure hoop strain. The strain was measured using an SR-4 strain indicator for the elastic studies.

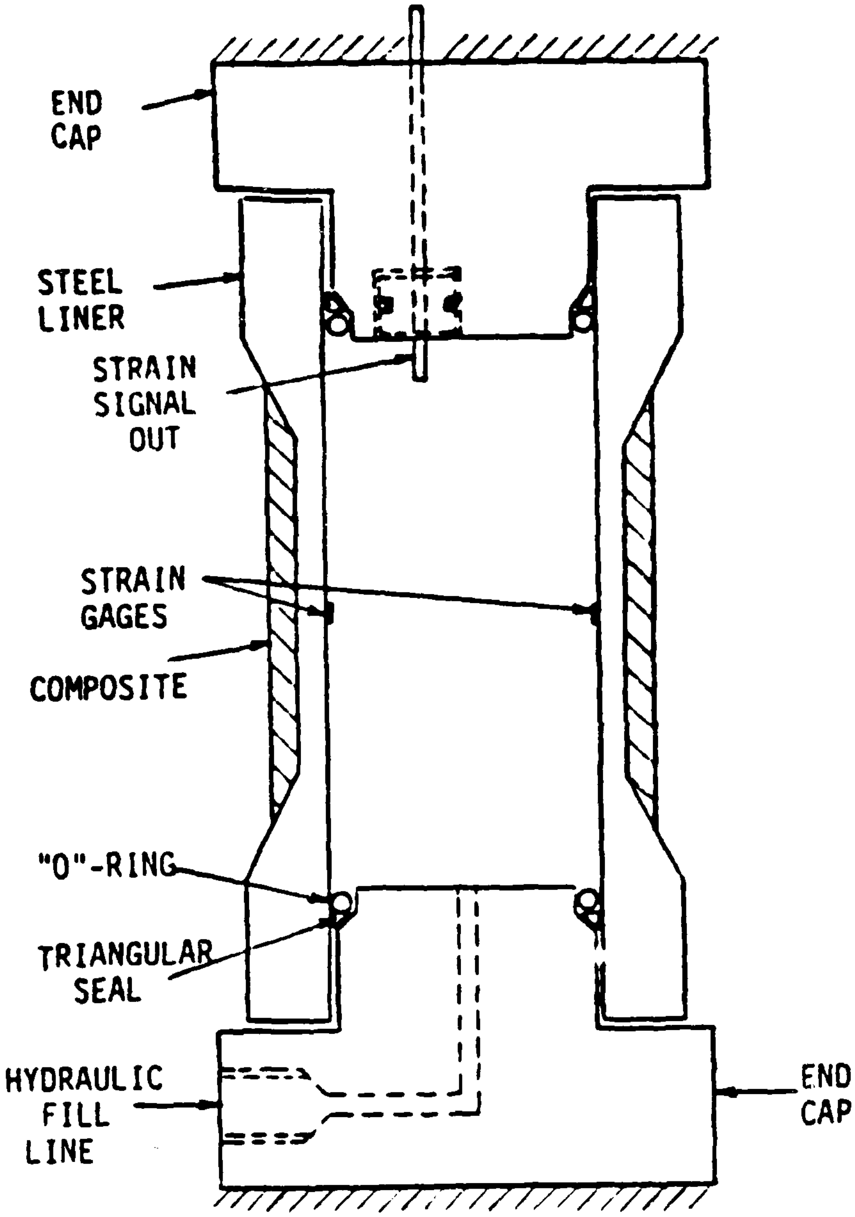

Figure 1. Schematic of the test setup used to measure bore strains in compound cylinders.

TABLE I. COMPARISON OF THEORETICAL AND EXPERIMENTAL STRAINS FOR COMPOUND CYLINDERS

W_{jl} (Total Wall Ratio)	Total Wall Thickness Composite %	Jacket and Liner Thickness in. (cm)	Liner Thickness in. (cm)	Jacket Thickness in. (cm)	Exp. Bore Strain Per Psi $\frac{\mu in./in.}{Psi}$ $\left(\frac{ncm/cm}{Pa}\right)$	Theoretical Bore Strain Per Psi $\frac{\mu in./in.}{Psi}$ $\left(\frac{ncm/cm}{Pa}\right)$
1.321	41	0.289 (0.734)	0.170 (0.432)	0.119 (0.304)	0.150 (0.0218)	0.147 (0.0213)
1.321	65	0.289 (0.734)	0.100 (0.254)	0.189 (0.480)	0.189 (0.0274)	0.163 (0.0237)
1.546	65	0.491 (1.247)	0.170 (0.432)	0.321 (0.815)	0.109 (0.0158)	0.116 (0.0168)
1.170	0	0.170 (0.432)	0.170 (0.432)	0 (0)	0.190 (0.0276)	0.181 (0.0263)

TEST SPECIMENS

Figure 2 is a schematic of the steel liner and Table I gives the details of the test specimen configuration. Three specimens were tested for comparison with theory. The specimens were constructed using steel liners with two thicknesses and the appropriate thickness of the composite circumferentially wound on the liner. Table II gives the material properties for the composite and the steel.

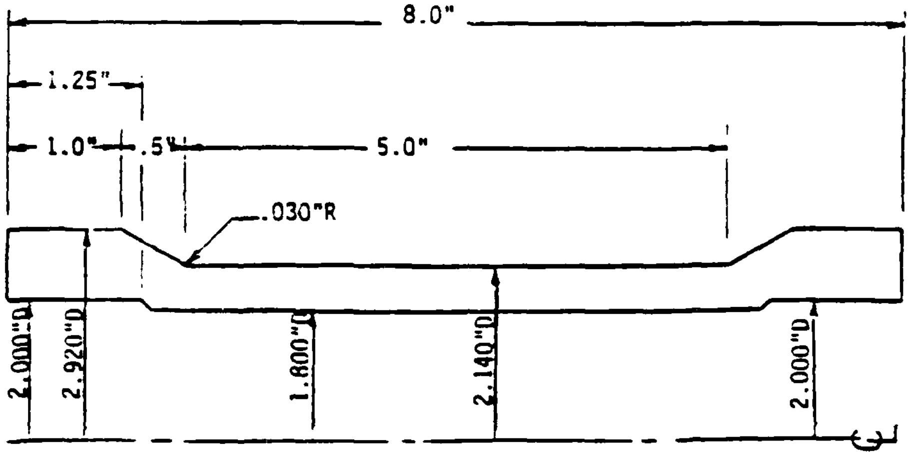

Figure 2. Schematic of the steel liner used in the compound cylinder tests.

TABLE II. MATERIAL PROPERTIES OF COMPOSITE JACKET AND STEEL LINER

Material Properties for IM6/Bismaleimide, 55% F.V.R.		
E_r = 1.126 MPsi (7.758 GPa)	$\nu_{r\theta}$ = 0.01524	$\nu_{\theta r}$ = 0.3155
E_θ = 23.31 MPsi (160.61 GPa)	$\nu_{\theta z}$ = 0.3155	$\nu_{z\theta}$ = 0.01524
E_z = 1.126 MPsi (7.758 GPa)	ν_{zr} = 0.3991	ν_{rz} = 0.3911

Material Properties for Steel		
E_s = 30.0 MPsi (206.7 GPa)	ν_s = 0.3	

ANALYSIS

General

The object of this investigation is to provide the designer with a method of replacing a steel (isotropic) cylinder with a compound cylinder. The concern of the designer is to maintain a bore stress, bore strain, and radial displacement in the replacement cylinder equal to that which existed in the original cylinder. The theoretical solution presented here permits calculating such information. The experimental results, which agree closely with the theoretical results, support the theory. The theoretical results show where a replacement compound cylinder can be used and also can predict a number of combinations of steel and composite thicknesses that satisfy the requirements for the all-steel isotropic cylinder. From the theoretical solution, a prediction can be made of the weight savings that results from using an equivalent compound cylinder.

Isotropic Cylinder Analysis

The designer uses the following equations to establish his design of an all-steel (isotropic) cylinder. The hoop strain at the bore of a steel cylinder subjected to internal pressure p and external pressure q is given by

Plane-stress boundary conditions:

$$\epsilon_{\theta,a} = \frac{1}{E_s} (\sigma_{\theta,a} - \nu_s \sigma_{r,a}) \tag{1}$$

Plane-strain boundary conditions:

$$\epsilon_{\theta,a} = \frac{1}{E_s} [\sigma_{\theta,a}(1-\nu_s^2)-\nu_s\sigma_{r,a}(1+\nu_s)] \tag{2}$$

where from Lamé's solution (ref 3) (for a cylinder of inside radius a and an outside radius b_o) $\sigma_{\theta,a}$ (the hoop stress at r = a) is given by

[3]S. P. Timoshenko and J. N. Goodier, Theory of Elasticity, McGraw-Hill, New York, 1970, pp. 68-71.

$$\sigma_{\theta,a} = p\,\frac{(b_o^2+a^2) - 2qb_o^2}{(b_o^2-a^2)} \tag{3a}$$

By introducing the parameter $W_s = b_o/a$, we have

$$\sigma_{\theta,a} = \frac{p(W_s^2+1) - 2qW_s^2}{(W_s^2-1)} \tag{3b}$$

and $\sigma_{r,a}$ (the radial stress at $r = a$) is given by

$$\sigma_{r,a} = -p \tag{4}$$

The radial displacement at the bore is simply the bore hoop strain equation (Eqs. (1) or (2)) times the bore radius (a) or

$$\delta_{r,a} = \epsilon_{\theta,a} \cdot a \tag{5}$$

These relations then give the solutions for the isotropic case. The experiment will give the results in terms of $\epsilon_{\theta,a}/p$ for a compound cylinder. These results can be converted to $\sigma_{\theta,a}$ and $\delta_{r,a}$, and thus, the effect of removing steel and replacing it with a composite jacket may be seen for the particular cases examined. The theoretical solution which follows allows the selection of many compound cylinders that meet the requirements of the original isotropic cylinder.

Compound Cylinder Analysis

The equations that represent the distribution of stress in a compound cylinder are obtained by combining Lamé's solution for an isotropic cylinder (ref 3) and Lekhnitskii's solution for an orthotropic cylinder (ref 2). The combination of these two solutions requires that two boundary conditions be satisfied at the steel-composite interface. The first is an equilibrium

[2]G. P. O'Hara, "Some Results on Orthotropic High Pressure Cylinders," ARCCB-TR-87015, Benet Weapons Laboratory, Watervliet, NY, June 1987.
[3]S. P. Timoshenko and J. N. Goodier, Theory of Elasticity, McGraw-Hill, New York, 1970, pp. 68-71.

condition that states: the radial stress at the material interface must be the same for both the steel and the composite. This would correspond to the external pressure in Lamé's equation for the steel being equal to the internal pressure of Lekhnitskii's equation for the composite (see Figure 3 for the decomposition of the compound cylinder problem into its two parts). The next condition requires steel-composite hoop strain equivalence at the material interface, i.e., the steel and the composite move together at the interface.

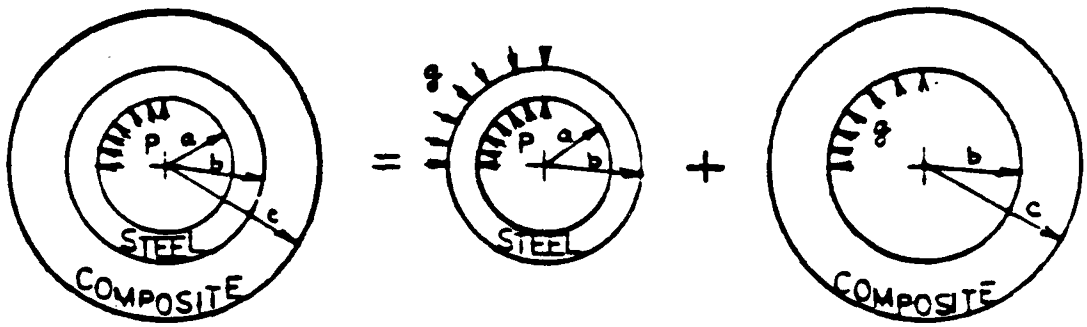

Figure 3. Decomposition of compound cylinder problem.

Similar to Eqs. (1) and (2), the equations which predict the hoop strain at the outside ($r = b$) of a steel cylinder subjected to internal pressure p and external pressure q are as follows:

Plane-stress boundary conditions:

$$\epsilon_{\theta,b} = \frac{1}{E_s} (\sigma_{\theta,b} - \nu_s \sigma_{r,b}) \tag{6}$$

Plane-strain boundary conditions:

$$\epsilon_{\theta,b} = \frac{1}{E_s} [\sigma_{\theta,b}(1-\nu_s^2) - \nu_s \sigma_{r,b}(1+\nu_s)] \tag{7}$$

where, from Lamé's solution (ref 3) (for inside radius a and outside radius b), $\sigma_{\theta,b}$ (the hoop stress at $r = b$) is given by

[3]S. P. Timoshenko and J. N. Goodier, Theory of Elasticity, McGraw-Hill, New York, 1970, pp. 68-71.

$$\sigma_{\theta,b} = \frac{2a^2p - q(a^2+b^2)}{(b^2-a^2)} \tag{8a}$$

By introducing the parameter $W_1 = b/a$, we have

$$\sigma_{\theta,b} = \frac{2p - q(W_1^2+1)}{(W_1^2-1)} \tag{8b}$$

and $\sigma_{r,b}$ (the radial stress at $r = b$) is given by

$$\sigma_{r,b} = -q \tag{9}$$

Similarly, the equation which predicts the hoop strain at the inside ($r = b$) of a composite cylinder subjected to internal pressure q only, is the following:

Plane-strain boundary conditions:

$$\epsilon_{\theta,b} = \frac{\sigma_{\theta,b}}{E_\theta} - \frac{\nu_{r\theta}\sigma_{r,b}}{E_r} - \frac{\nu_{z\theta}\sigma_{z,b}}{E_z} \tag{10}$$

where from Lekhnitskii's solution (ref 2) (for inside radius of b and outside radius of c), $\sigma_{\theta,b}$ (the hoop stress at $r = b$) is given by

$$\sigma_{\theta,b} = \frac{qK[(b/c)^{2K} + 1]}{1 - (b/c)^{2K}} \tag{11a}$$

By introducing the parameter $W_j = c/b$, we have

$$\sigma_{\theta,b} = qK \frac{[W_j^{2K} + 1]}{[W_j^{2K} - 1]} \tag{11b}$$

$\sigma_{r,b}$ (the radial stress at $r = b$) is given by

$$\sigma_{r,b} = -q \tag{12}$$

and $\sigma_{z,b}$ (the axial stress at $r = b$) is given by

$$\sigma_{z,b} = -q\left[\frac{\nu_{rz}E_z}{E_r} - \frac{K\nu_{z\theta}(W_j^{2K}+1)}{(W_j^{2K}-1)}\right] \tag{13a}$$

[2]G. P. O'Hara, "Some Results on Orthotropic High Pressure Cylinders," ARCCB-TR-87015, Benet Weapons Laboratory, Watervliet, NY, June 1987.

Also, by comparing $\sigma_{z,b}$ with Eq. (11b) and Eq. (12), we have

$$\sigma_{z,b} = \nu_{rz} \frac{E_z}{E_r} \sigma_{r,b} + \nu_{z\theta}\sigma_{\theta,b} \tag{13b}$$

By substituting Eq. (13b) into Eq. (10) and rearranging, a more concise form of Eq. (10) can be obtained:

$$\epsilon_{\theta,b} = \frac{\sigma_{\theta,b}}{E_\theta} (1 - \nu_{\theta z}\nu_{z\theta}) - \frac{\sigma_{r,b}}{E_r} (\nu_{r\theta} + \nu_{rz}\nu_{z\theta}) \tag{14}$$

Note the similarity in the form of Eqs. (14) and (7). The orthotropic-material parameter K can be expressed in a concise form as

$$K = \sqrt{\frac{\dfrac{E_z}{E_r} - \nu_{zr}^2}{\dfrac{E_z}{E_\theta} - \nu_{z\theta}^2}} \tag{15}$$

By equating $\epsilon_{\theta,b}$ of the steel (Eqs. (6) and (7)) with $\epsilon_{\theta,b}$ of the composite (Eq. (14)), we can express the interface pressure q as a function of steel internal pressure p, steel-composite material properties, and the radii a,b,c. In a form that is non-dimensional with respect to geometry, the expression for q reduces to the following:

$$q = \frac{2p}{(W_1^2-1)\left[AK \dfrac{(W_j^{2K}+1)}{(W_j^{2K}-1)} + B + \dfrac{(W_1^2+1)}{(W_1^2-1)}\right]} \tag{16}$$

The two constants A and B are given by

Plane-stress boundary conditions for steel:

$$A = \frac{E_s}{E_\theta}\left[1 - \nu_{\theta z}\nu_{z\theta}\right] \tag{17}$$

$$B = E_s\left[\frac{(\nu_{r\theta} + \nu_{rz}\nu_{z\theta})}{E_r} - \frac{\nu_s}{E_s}\right] \tag{18}$$

and for

Plane-strain boundary conditions for steel:

$$A = \frac{E_s}{E_\theta}\left[1 - \nu_{\theta z}\nu_{z\theta}\right]\frac{1}{(1-\nu_s^2)} \tag{19}$$

$$B = E_s\left[\frac{(\nu_{r\theta} + \nu_{rz}\nu_{z\theta})}{E_r} - \frac{\nu_s(1+\nu_s)}{E_s}\right]\frac{1}{(1-\nu_s^2)} \tag{20}$$

By comparing Eqs. (6) and (7) with Eq. (14), the origin for the terms that make up the A and B constants becomes apparent. It should also be noted that the case of generalized plane-strain (zero net axial force) for the composite has been neglected in this report because the equations which define the stress distribution in the composite jacket for this case are very complicated. It has been observed, however, that for a carbon bismaleimide system with the fibers in the hoop direction, the difference between the plane-strain solution and the generalized plane-strain solution results in an error of less than one percent in q. This is because the low axial stiffness of an all-hoop layup allows for only a very small axial stress when an axial constraint is enforced.

The equation for q (Eq. (16)) can now be substituted back into the more general equations for stress (refs 2,3) to obtain the complete distribution in the steel liner and the composite jacket. From the complete stress distribution, the hoop strain at any point in the liner and jacket can be calculated.

The hoop strain at the bore of the steel liner is an important value to use in assessing the effectiveness of the composite as a reinforcement. Using a carbon-based fiber bismaleimide system for a composite jacket results in a more compliant cylinder than if it were all steel. The reason for this is that the

[2]G. P. O'Hara, "Some Results on Orthotropic High Pressure Cylinders," ARCCB-TR-87015, Benet Weapons Laboratory, Watervliet, NY, June 1987.
[3]S. P. Timoshenko and J. N. Goodier, Theory of Elasticity, McGraw-Hill, New York, 1970, pp. 68-71.

low radial stiffness of the composite does not allow effective transfer of load through the wall thickness of the composite. If we define a parameter percent composite equal to $\frac{(c-b)}{(c-a)}$ x 100, we can determine how the liner's bore hoop strain varies as a function of overall wall ratio W_{jl} for various percent composites.

Now that we can determine the stress and strain distribution in a compound cylinder, it is desirable to determine how much weight can be saved by using a composite jacket. In making this assessment of weight savings, we will compare an all-steel cylinder of inner radius a with a compound cylinder of the same inner radius. It will also be required that the bore hoop strain per unit of internal pressure for both cylinders must be equal. This is done by using Eqs. (1) or (2) for the hoop strain at the bore. For the all-steel cylinder subjected to internal pressure only, the q in Eq. (3) is equal to zero. For the compound cylinder, it is necessary to replace the b_o found in Eq. (3) with b and to use the expression for q found in Eq. (16). The final result of carrying out the necessary algebraic manipulation to satisfy the above constraints is an expression for the wall ratio of the composite jacket W_j.

$$W_j = \left[\frac{KA + \frac{(1+R^2)}{(1-R^2)} - B}{\frac{(1+R^2)}{(1-R^2)} - B - KA} \right]^{\frac{1}{2K}} \tag{21}$$

where $R = W_l/W_s$.

The interesting aspect of Eq. (21) is that W_j is dependent only on material properties and the parameter R. This expression for W_j defines the amount of composite that is needed to give the compound cylinder an equivalent radial stiffness to that of the all-steel cylinder.

The weight per unit length of the all-steel cylinder is given by

$$WGHT_s = \rho_s \pi (b_o^2 - a^2) \tag{22}$$

The weight per unit length of the equivalent compound cylinder is given by

$$WGHT_{j1} = \pi \rho_s (b^2 - a^2) + \pi \rho_j (c^2 - b^2) \tag{23}$$

where ρ_s = density of steel, and

ρ_j = density of composite jacket

The percent weight reduction by using the compound cylinder to replace a steel cylinder is

$$\% \, WR = \left[\frac{WGHT_s - WGHT_{j1}}{WGHT_s} \right] \cdot 100 \tag{24}$$

This reduces to

$$\% \, WR = \left[1 - \frac{(W_1^2 - 1) + W_1^2 (W_j^2 - 1)/F}{(W_s^2 - 1)} \right] \cdot 100 \tag{25}$$

where $F = \rho_s / \rho_j$.

Now, by using Eq. (25) with Eq. (21) and by noting that the overall compound cylinder wall ratio (W_{j1}) is defined as

$$W_{j1} = W_j \cdot W_1 \tag{26}$$

it is possible to determine the % WR as a function of W_{j1} for a specific value of all-steel wall ratio (W_s).

It can also be shown that for a given W_s, there is a single value of W_1 that makes the % WR (Eq. (25)) a maximum. This value of W_1 also determines a value of R ($R = W_1/W_s$). By setting the derivative of Eq. (25) with respect to R equal to zero, we can obtain an expression that will predict the % WR and geometry of the lightest possible equivalent compound cylinder to replace an all-steel cylinder of wall ratio W_s.

The final result of maximizing Eq. (25) gives the following:

$$F = (1 - RW_j \frac{dW_j}{dR} - W_j^2) \tag{27}$$

Since F is a known material parameter ρ_s/ρ_j, and since W_j is a function of R and material properties, the one real value of R which solves Eq. (27) can be found by iteration. The equation defining $\frac{dW_j}{dR}$ is given by

$$\frac{dW_j}{dR} = \frac{-4RA}{(1-R^2)^2} \frac{\left[KA + \frac{(1+R^2)}{(1-R^2)} - B\right]^{(\frac{1}{2K} - 1)}}{\left[\frac{(1+R^2)}{(1-R^2)} - B - KA\right]^{(\frac{1}{2K} + 1)}} \tag{28}$$

DISCUSSION OF RESULTS

The purpose of this effort is to present a concise method of designing compound cylinders which have characteristics at the inside diameter that are equal to those of an all-steel cylinder. The results from the experimental efforts on compound cylinders are compared to the appropriate all-steel cylinder. The experimental results are also compared with the theoretical results to support the theory and allow its extension to the most general design case. Finally, the weight savings are presented for the particular cases.

The experimental measurements were made on the three specimens previously described. The data are in terms of pressure versus bore hoop strain. Figures 4a and 4b are presentations of the experimental data for the three cylinders tested. Figure 4a presents the data for the 1.321 wall ratio (W_{j1}) and Figure 4b presents the data for the 1.546 wall ratio (W_{j1}). Included in both of these figures is the isotropic (all-steel) solution for plane-stress boundary conditions, given by Eq. (1). Figure 4a shows that as steel is removed and

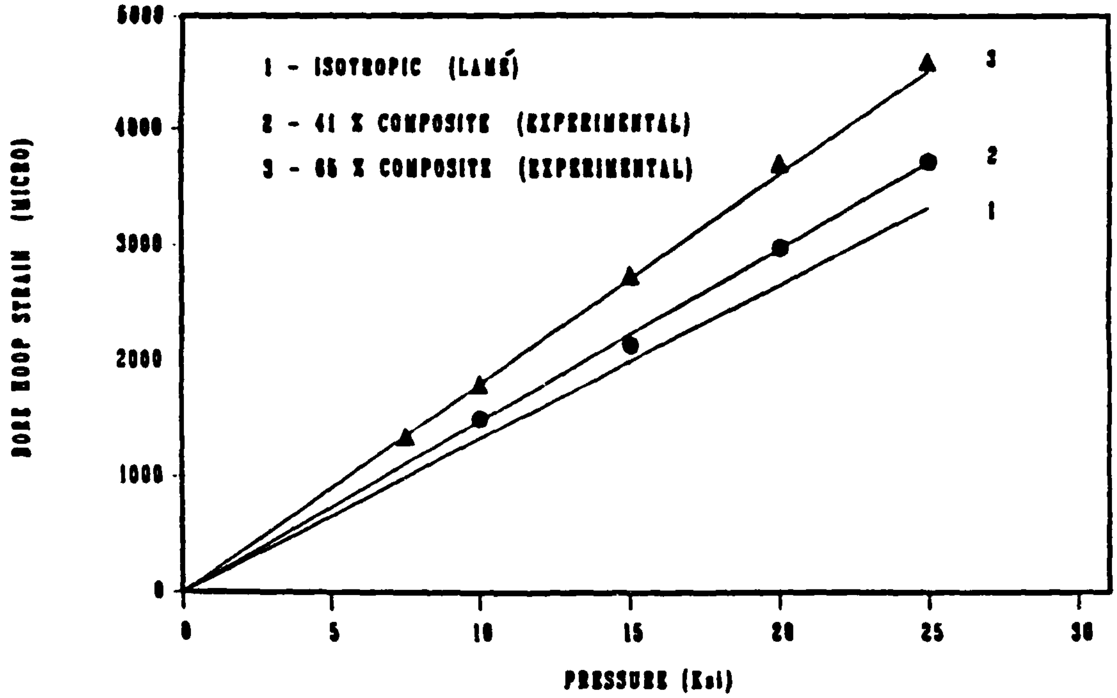

Figure 4a. Experimental bore hoop strain for a compound cylinder of $W_{j1} = 1.321$.

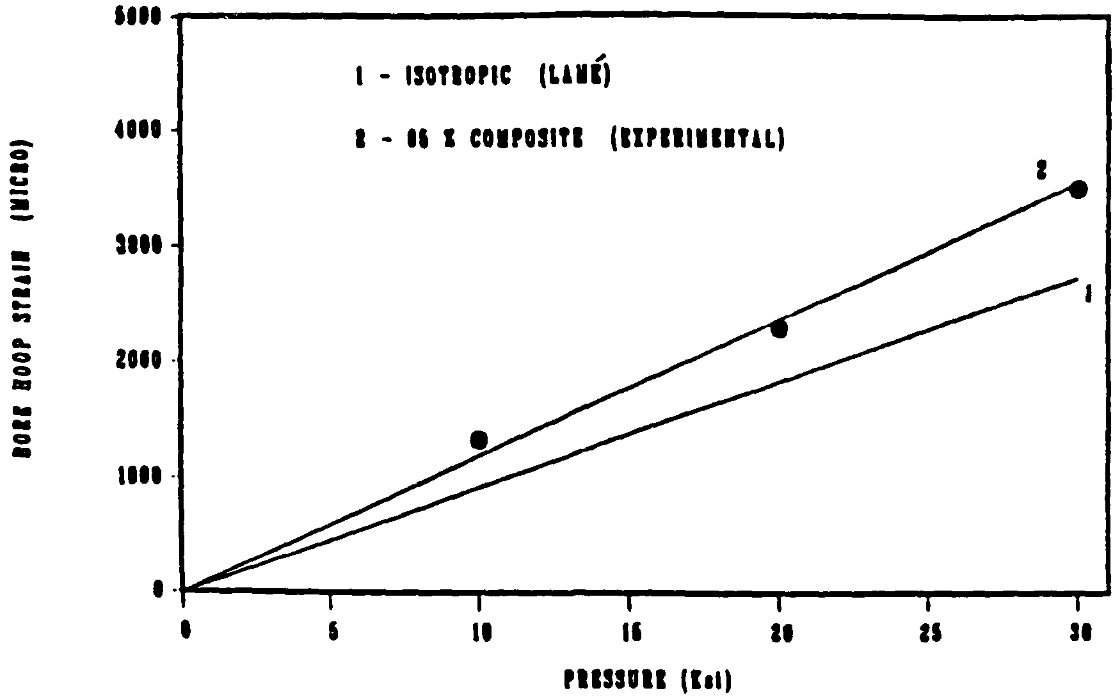

Figure 4b. Experimental bore hoop strain for a compound cylinder of $W_{j1} = 1.546$.

replaced by composite material, the bore hoop strain increases for the same internal pressure. This difference is also shown in Figure 4b for the second wall ratio. In order to maintain the same bore hoop strain for a given internal pressure in a compound cylinder, the wall ratio must increase. Table I presents the experimental results and some results from the theoretical investigation presented in the Compound Cylinder Analysis section of this report. Comparing the experimental and theoretical bore hoop strains shows they are in good agreement. Figure 5 is a theoretical plot of bore hoop strain ($\epsilon_{\theta,a}$) versus overall wall ratio (W_{jl}) for cylinders that vary from fully isotropic (all-steel $W_{jl} = W_s$) to fully orthotropic (all-composite). The values of hoop strain in

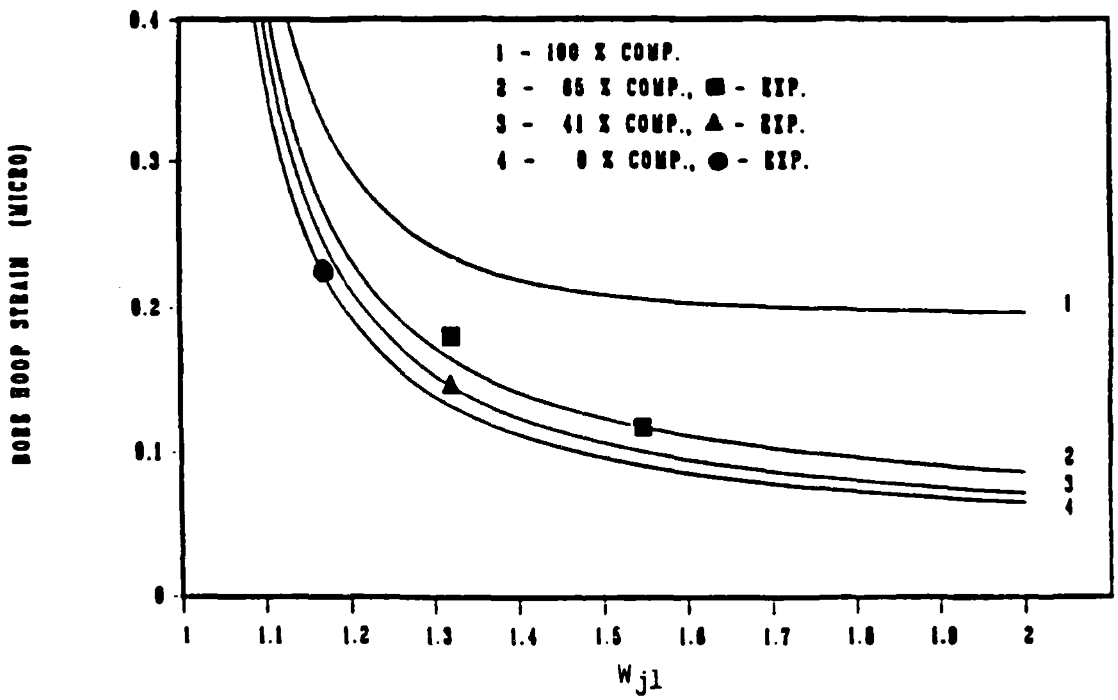

Figure 5. Theoretical results of bore hoop strain for various compound cylinders.

this figure are for plane-stress boundary conditions and are calculated from Eqs. (1), (3), and (4) using the q from Eq. (16) and replacing b_0 with b. The four experimental points are also plotted on this figure. Note that for a given W_{j1}, the curve representing the hoop strain at 65 percent composite is very close to the zero percent composite (fully isotropic) curve. This shows that for fairly high values of percent composite, the compound cylinder does not lose a significant amount of its radial stiffness when compared to the isotropic cylinder.

The four curves can be used to select a compound cylinder with the same bore characteristics as the isotropic cylinder. This is done by obtaining the bore hoop strain for the isotropic case and drawing a horizontal line at that value; all the compound cylinders that intersect that line have the same bore characteristics but with greater wall ratios. The larger wall ratio is not a detriment to the weight savings as the composite density is 20 percent the density of steel. Figure 6 is an example of the weight savings that might be achieved in a steel cylinder with a wall ratio of 1.255. At an overall wall ratio of approximately 1.35, the weight savings is 34 percent and the percent of the wall thickness that is composite material is 68 percent. The dimensions for the steel and compound cylinders with inner radius of 2.362 (6.0 cm) inches can best be compared in Table III. The maximum weight savings is discussed next, but it is interesting to note that a weight savings less than the maximum may be chosen to get a thicker section of steel. For instance, a thick section of steel may be of interest from the point of view of fatigue life. Also note that there is a particular value of W_{j1} that gives a maximum percent weight reduction. In this case, the maximum weight savings has a value of about 40 percent

TABLE III. COMPARISON OF DIMENSIONS AND WEIGHT SAVINGS FOR A COMPOUND CYLINDER AND AN EQUIVALENT STEEL CYLINDER

Material	Inner Radius in. (cm)	Outer Radius in. (cm)	W_{j1}	Wall Thickness in. (cm)	Steel Thickness in. (cm)	$\frac{\text{Cylinder Weight}}{\text{Steel Cylinder Weight}}$
Steel	2.362 (5.999)	2.965 (7.531)	1.255	0.60 (1.524)	0.60 (1.524)	1
Compound	2.362 (5.999)	3.189 (8.100)	1.35	0.83 (2.108)	0.27 (0.686)	0.66

and occurs at a W_{jl} of about 1.4. This means that the lightest equivalent compound cylinder, using this specific composite material to replace an all-steel cylinder of $W_s = 1.255$, has an overall wall ratio of about $W_{jl} = 1.4$.

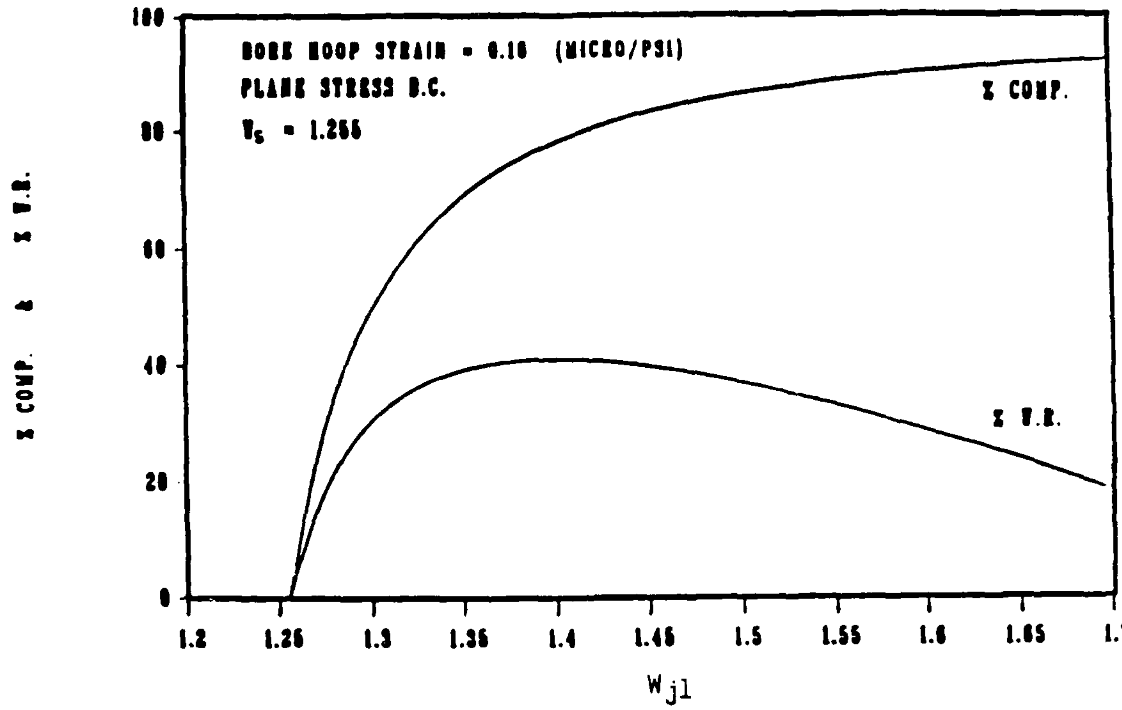

Figure 6. Theoretical results of percent composite and percent weight reduction for compound cylinders equivalent to an all-steel cylinder of $W_s = 1.255$.

By using Eq. (27) in conjunction with Eqs. (28), (26), and (21), it is possible to determine the value of W_{jl} that gives a maximum percent weight reduction over an all-steel tube of wall ratio W_s. Figure 7 shows a plot of percent composite and maximum percent weight reduction versus all-steel cylinder wall ratio W_s. Both the plane-stress and plane-strain solutions for the steel are shown. It is interesting to note that there is only one value of R that maximizes the weight savings for the whole set of all-steel wall ratios W_s. Also from Eq. (21), the value of R corresponds to a single value of W_j. For the composite material used in this report, $F = \rho_s/\rho_j = 5.0$, and by iteration of Eq.

(27), R = 0.869 (plane stress), R = 0.881 (plane strain). Now substituting these values of R into Eq. (21), we find W_j = 1.282 (plane stress), W_j = 1.271 (plane strain). We can now calculate W_1 by noting $R = W_1/W_s$. Finally, W_{j1} can be easily obtained by using the relation $W_{j1} = W_j \cdot W_1$.

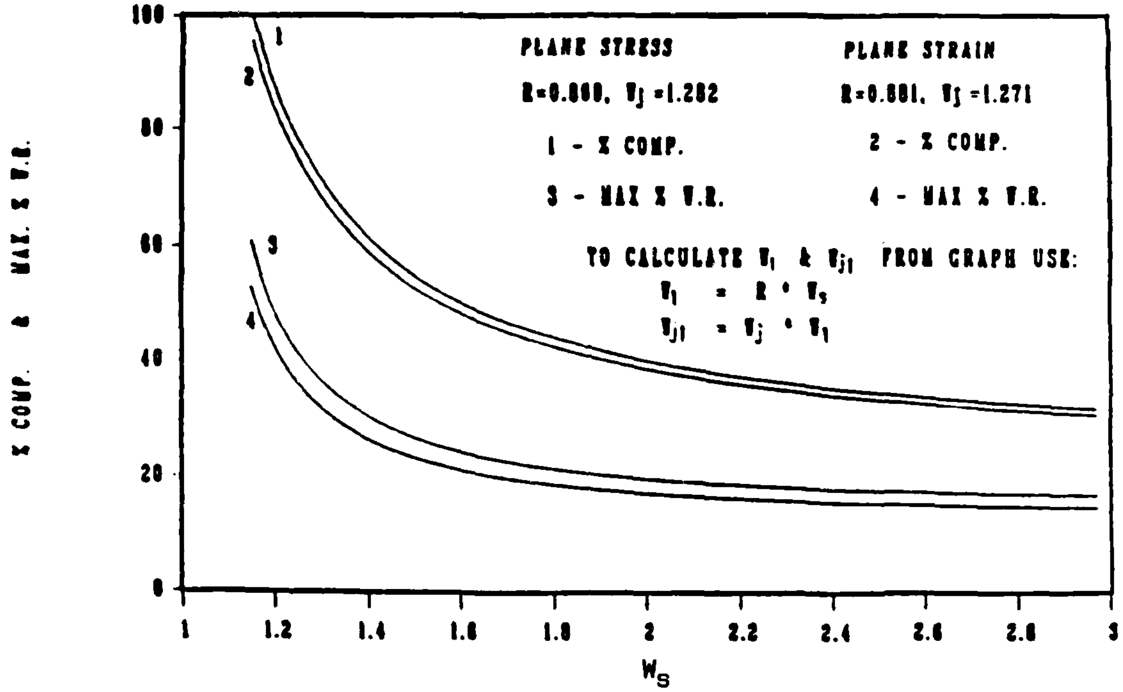

Figure 7. Theoretical predictions of lightest possible equivalent compound cylinders to replace all-steel cylinders.

CONCLUSIONS

The theoretical results presented allow the design of a compound cylinder having the same bore characteristics as a monoblock steel cylinder. The bore strains, stresses, and radial displacement are equal in both cylinders. The following specific conclusions can be made from the experimental and theoretical investigations:

1. As steel is removed from a cylinder and replaced by composite material, the bore hoop strain, stress, and radial displacement increase.

2. There are many compound cylinders with greater wall ratios than an all-steel cylinder that will have equivalent bore hoop strain, stress, and radial displacement.

3. An example was shown where the weight savings was 34 percent and the cylinder wall was 68 percent composite. The increase in outside diameter for this case was from 5.93 inches (15.1 cm) to 6.38 inches (16.2 cm).

4. A set of equations was presented that can be used to design compound cylinders and to calculate the weight savings.

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